Evolving Modes of Gravitational Perturbations in a Black Hole Space-Time

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Abstract

Gravitational waves are ripples in space-time created by accelerating objects. Detecting and analyzing these waves would aid in better understanding the systems which produce them. While all accelerating objects produce waves, most are too weak to detect. The overall intent of this research is to simulate binary black hole systems with extreme mass ratios, which are predicted to produce a signal strong enough to be detected by space-based gravitational wave detectors. In simulating this, we are able to get information on how the gravitational waves produced by such systems are expected to behave. The goal of this project was to evolve the gravitational perturbations of the orbiting black hole in order to move a step closer to truly simulating these systems.

Background

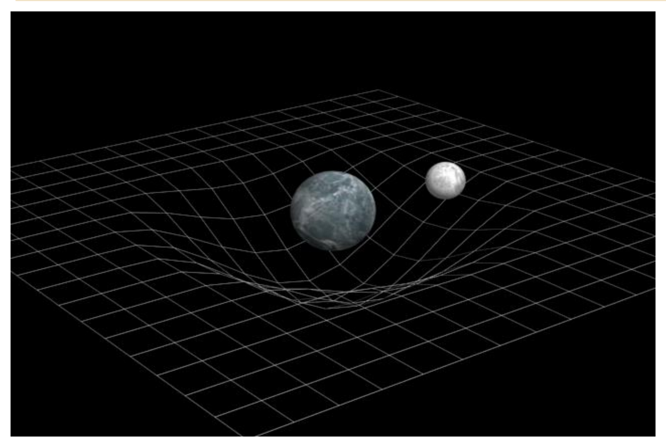

Figure 1. Einstein's theory of general relativity states that matter causes distortions in space-time, as depicted to the left. These distortions are felt as gravity.[1]

General relativity predicts the existence of gravitational waves (GWs), which are ripples in space-time that form when objects accelerate. One event which is likely to be a source of GWs measurable from planned space-based detectors is known as an extreme mass ratio inspiral (EMRI).

Extreme Mass Ratio Inspiral

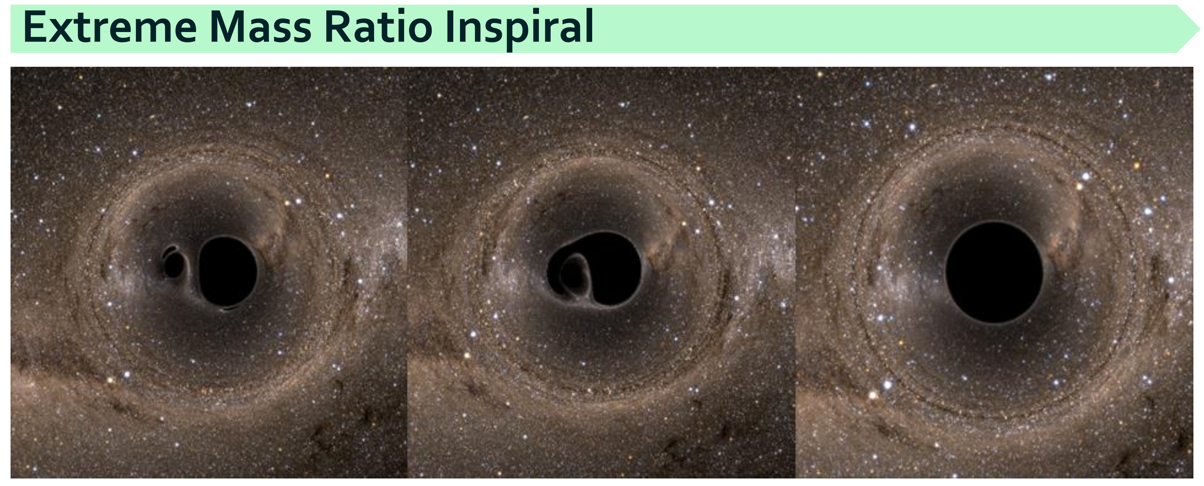

Figure 2. An EMRI is a system which consists of a low-mass object orbiting around a much heavier object. The theory predicts that as GWs carry energy and angular momentum away from the system, the lighter object will experience orbital decay, spiral inward and eventually merge with the central object. In this project both objects are black holes, as illustrated above. [2]

Perturbation Theory

To simulate a black hole EMRI, one must solve Einstein's field equations to get a space-time metric. In this case the field equations are too complex to solve exactly, so instead perturbation theory was used. According to this theory, when some small parameter is present in the problem one can approximate a solution by adding small perturbations to some related background solution. In this case, the small parameter is the mass ratio of the smaller to larger object, the background solution is the Schwarzschild metric for a non-rotating black hole, and the perturbations are produced by the orbiting black hole.

Previous Work

Coming into this project, the existing code was able to evolve a scalar field on the background space-time of the Schwarzschild black hole. The current work sought to evolve the metric perturbations.

Field Equations

$$\frac{\partial^2 h^i}{\partial t^2} = c_1 \frac{\partial^2 h^i}{\partial r^2} + c_2 \frac{\partial^2 h^i}{\partial t \delta r} + c_3 \frac{\partial h^i}{\partial r} + c_4 \frac{\partial h^i}{\partial t} + M^i_j h^j$$

$$M^3_j h^j = -\frac{f}{2r^2}\left(h^1 - h^5 - \left(1 - \frac{4M}{r}\right)(h^3 + h^6)\right)$$

$$M^9_j h^j = \frac{f}{r^2}\left(1 - \frac{9M}{2r}\right)h^9 - \frac{f}{2r^2}(1 - \frac{3M}{r})h^{10}$$

Output

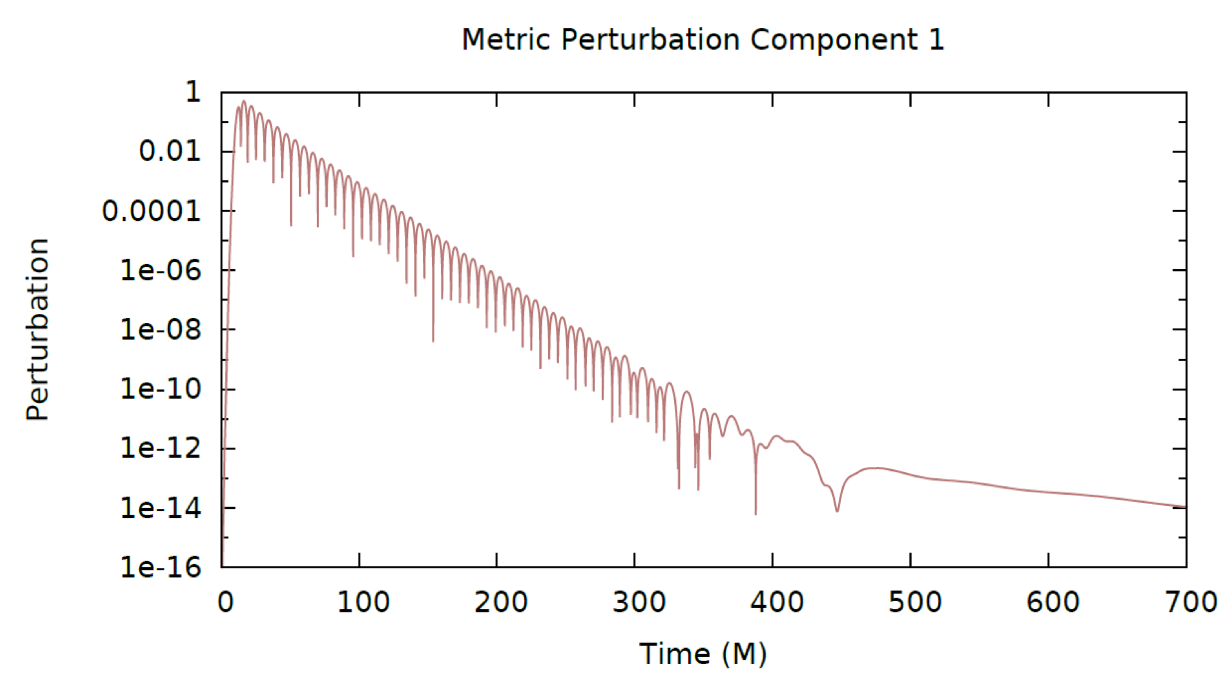

Figure 3

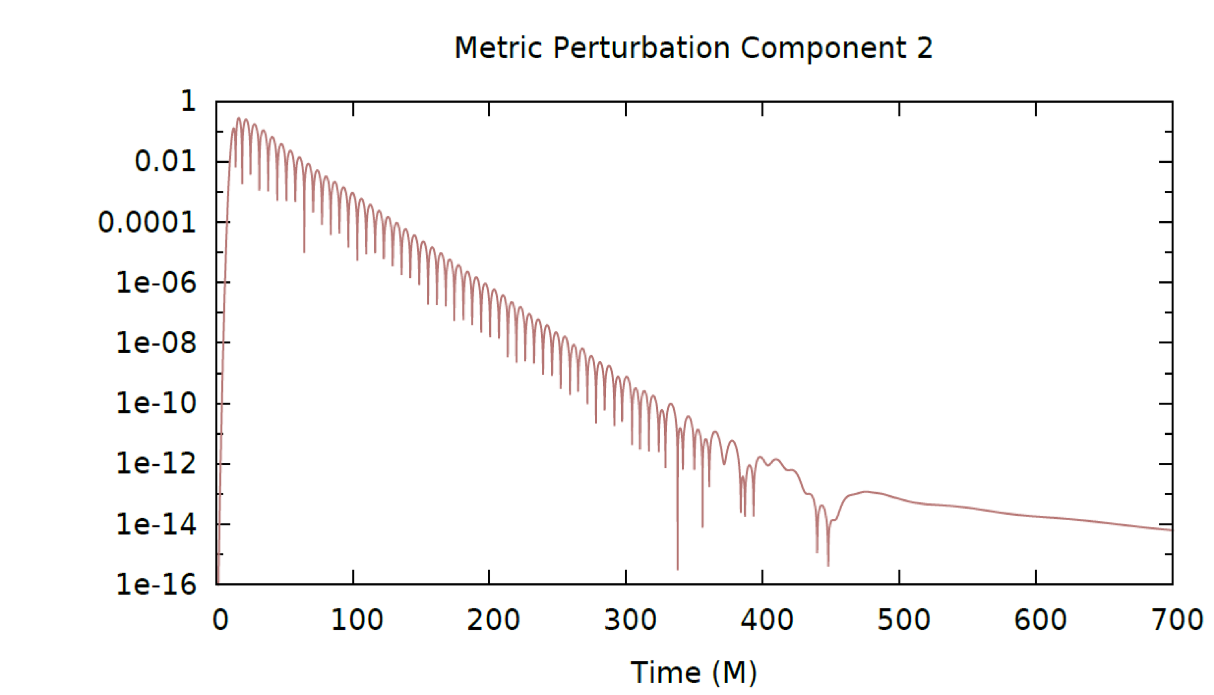

Figure 4

Description

To the left are the 10 harmonically decomposed field equations for this system.[3,4] Only two of ten coupling terms are shown. Components 1-7 couple and 8-10 couple with each other.

- h^i: perturbation components
- M^i_j: coupling matrix
- i,j: index locations within h^i and M^i_j
- c_1 - c_4 : spatial-dependent coefficients
- r: spatial coordinate
- t: time coordinate
- M: mass of the central black hole
- f = 1-2M/r

Discussion

Description

- Figures 3 and 4 show the absolute value of the perturbations calculated by the program for the l = 2, m=0 modes of components 1 and 2, plotted logarithmically.
- The unit of time, M, is the amount of time required for light to travel the radius of the large black hole.
- An initial pulse was given to component 1 of the metric.
- Components 1-7 are then excited through coupling.
- Since components 8, 9, and 10 do not couple with 1-7, they were not excited and remained at 0.
- Every component which is excited by another will have a smaller initial amplitude, as seen in figures 3 and 4.

Results

- The perturbations are shown to evolve as waves, as expected.
- The perturbations should also exponentially decay in amplitude initially as some parts of the wave travel through the large black hole's event horizon and others out to infinity. This is also produced by the program.
- As the waves reflect off of the curvature of the background space-time, the amplitude goes into a power law decay as expected.
- In order to get the correct tail behavior, coordinate transformations must be made for regions of space near the event horizon and approaching infinity. Since these have not yet been completed, the lack of smoothness in the tail of the graph is expected.

Significance

A simulation of systems which produce gravitational waves is particularly important since we have yet to detect them. Simulations give information on how both the waves and the systems themselves behave and evolve. Gravitational wave analysis can also provide a better understanding of the energy, evolution, and space-time geometry of the systems which produce them.

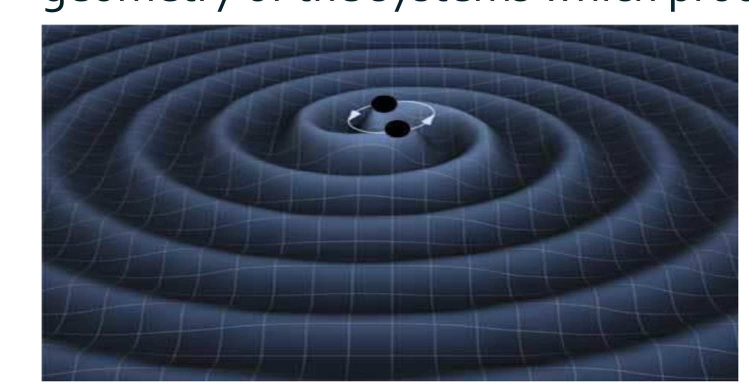

Figure 5. A binary black hole system producing gravitational waves.[5]

Future Work

In order to create a true simulation of a black-hole EMRI, some additional changes must be made. An effective source must be implemented to act as the source of the gravitational perturbations. Also, currently it is known that the equations used here are written in a coordinate system which simplifies them, but also does not give correct results in the l=0 and l=1 harmonic modes. So, some solution must be found to work with these modes.

Acknowledgements

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1) *Spacetime curvature*. (n.d.). Retrieved from Wikimedia Commons: https://commons.wikimedia.org/wiki/File:Spacetime_curvature.png

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