# Evolving Scalar Fields on a Hyperboloidal Slicing

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## Background

- Gravitational Waves (GW) are created when masses are accelerated
- A prominent (measurable) source of GW comes from a Black Hole (BH) binary system.
- GW produced from an accelerating BH experiences partial reflection due to the curvature of space.
- Reflected GW return to the BH and cause a change to the BH's orbit (Fig. 1).
- The effect on the BH due to the back-scattered GW is known as the self-force.

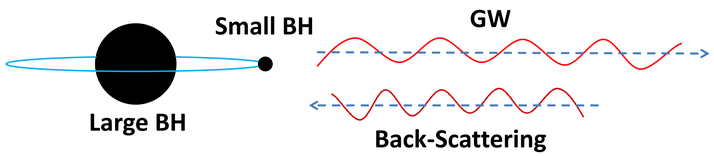



**Fig. 1:** An image showing the emitted GW from the BH binary and the reflected GW.

- Simulations of extreme mass ratio BHs require a significant amount of computation where a portion is dedicated to evolving GW.
- In order to determine the self-force on the smaller BH, the emitted GW is evolved towards future null infinity to account for the total reflected wave.
- This project explores an **efficient method** that evolves 1D scalar fields to future null infinity [1].

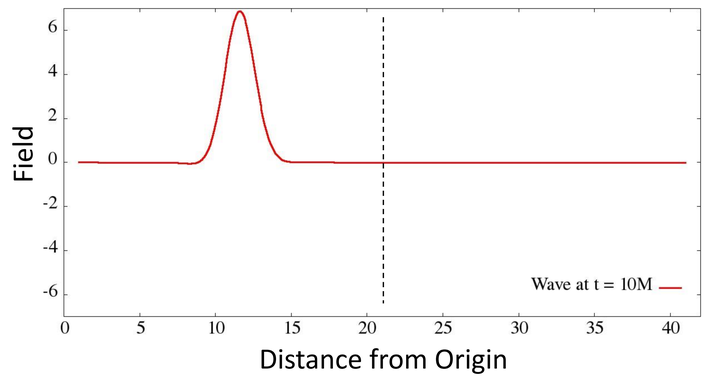
## Program Overview

- A pulse, represented by a Gaussian function, is modeled using the wave equation.
- By imposing boundary conditions and implementing coordinate transformations, the pulse travels from flat space onto a hyperboloidal slicing.
- The hyperboloidal foliation sets the outer boundary to future null infinity through space-time compactification.

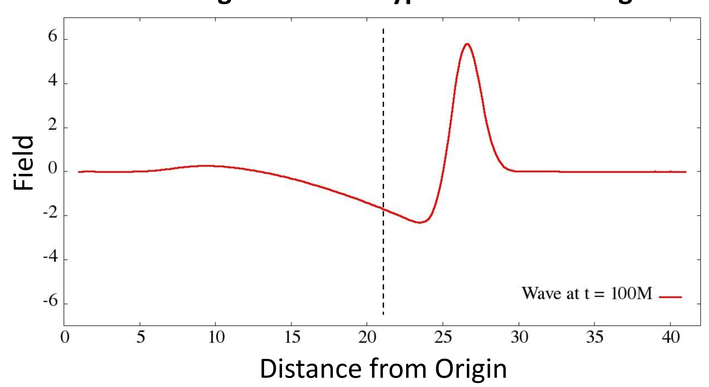
## Scalar Field Evolution

- A pulse travels smoothly from flat space onto a hyperboloidal slicing.
- The pulse appears to propagate cleanly off the grid (as expected) when the term in the wave equation that takes into account the angular structure of the pulse is neglected.
- Unfortunately, by including the angular component, the amplitude of the wave suddenly grows at future null infinity (not shown).
- Small fluctuations are visible at the outer boundary (future null infinity).
- Plots/animations for the evolution of a scalar wave transitioning from spatial coordinates (left) to a hyperboloidal slicing (right) are shown below.

**Evolving Pulse on a Hyperboloidal Slicing**

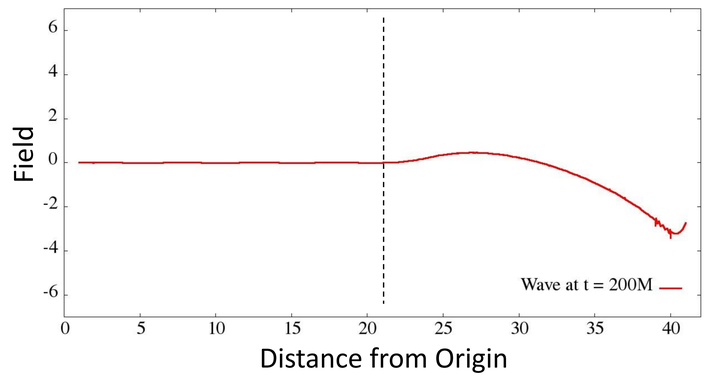



**Evolving Pulse on a Hyperboloidal Slicing**

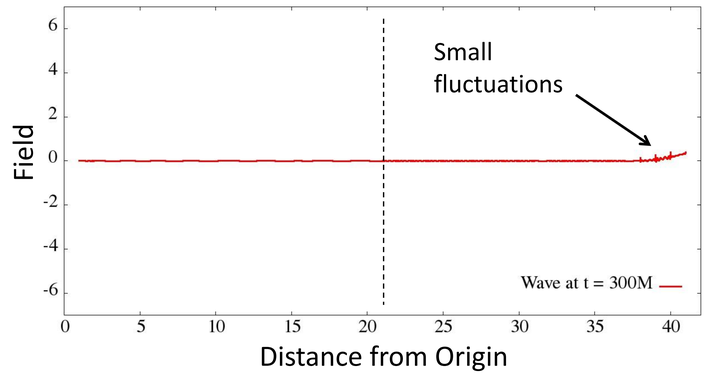



## Evolution Cont.

**Evolving Pulse on a Hyperboloidal Slicing**




**Evolving Pulse on a Hyperboloidal Slicing**




## Future Work

- Further test and evaluate outer boundary conditions to remove observed instabilities.
- Implement transition from Schwarzschild metric to hyperboloidal slicing.
- Integrate coordinate transformation method in extreme mass ratio BH simulations in order to **reduce computation time**.

**References:** [1] Bernuzzi Sebastiano, et al. *Binary black hole coalescence in the large-mass-ratio limit: the hyperboloidal layer method and waveforms at null infinity*. Phys. Rev. D. 12 Oct. 2011.

**Acknowledgements:** This material is based upon work supported the National Science Foundation under award OCI-1263236 with additional support from the Center for Computation & Technology at Louisiana State University.