Evolving Scalar Field Waves in the Vicinity of a Black Hole

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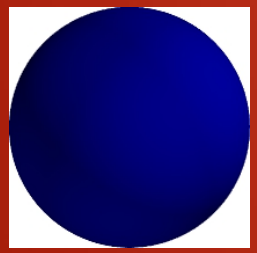
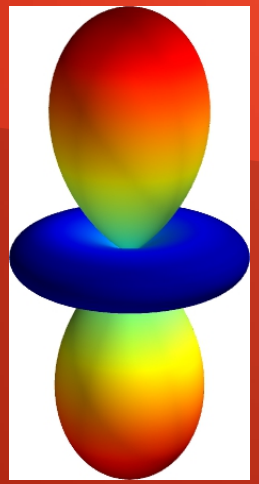
Introduction

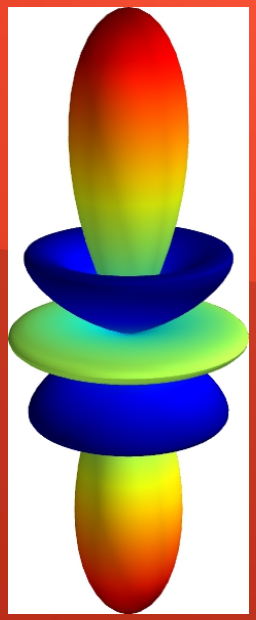
- The purpose of this project was to test a numerical method that could be used to run faster simulations of extreme mass ratio colliding black holes
- Although scalar field waves are not exactly like gravitational waves the program will provide a framework for more advanced simulations
- The program allows one to to test the method and analyze the problems that may arise from using this method

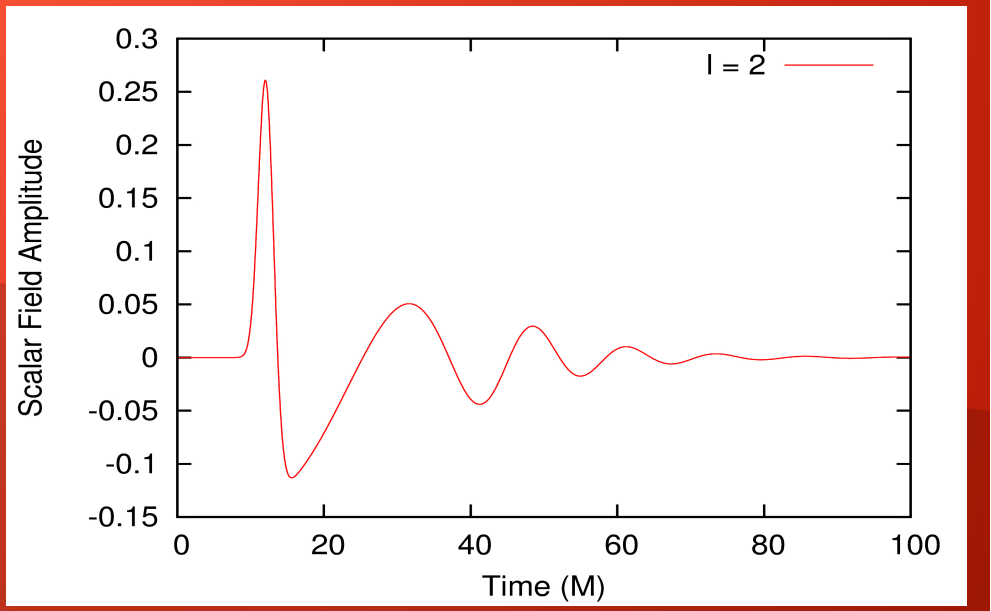
Methods

- The program uses the wave equation, $d^2u/dx^2 = d^2u/dt^2$, to model a scalar wave
- Values of u, du/dx, and du/dt are stored
- Several numerical methods such as finite differencing, characteristic variables, and Runge-Kutta are used to calculate each next time-step
- For the Kerr case, the program must also keep track of several different modes of the spherical harmonic
- The different modes are “coupled” by using the two nearest modes in the calculation of a mode's time derivative.
- An uncoupled version of the program is also used to compare accuracy



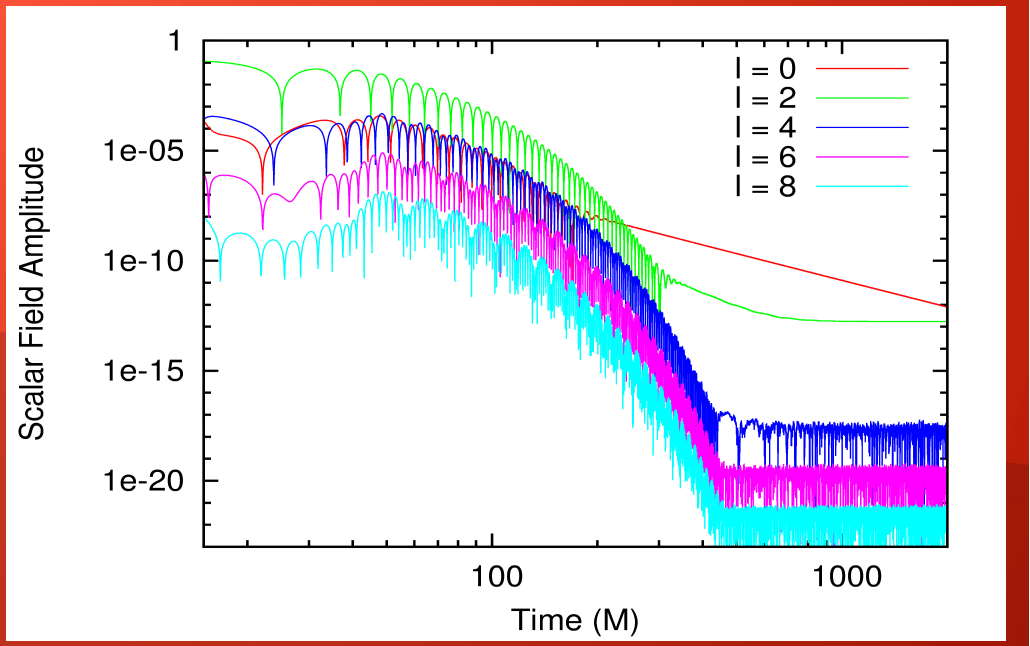






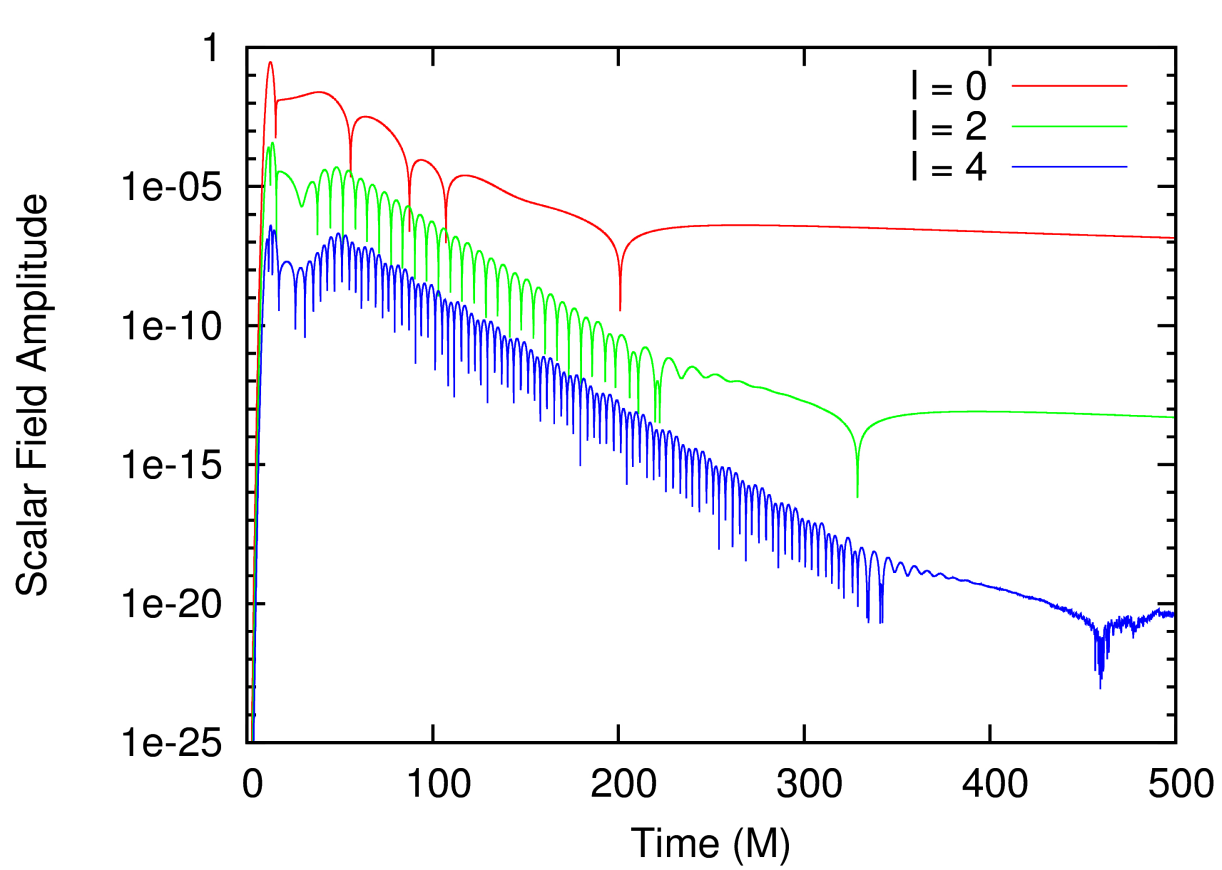
Results

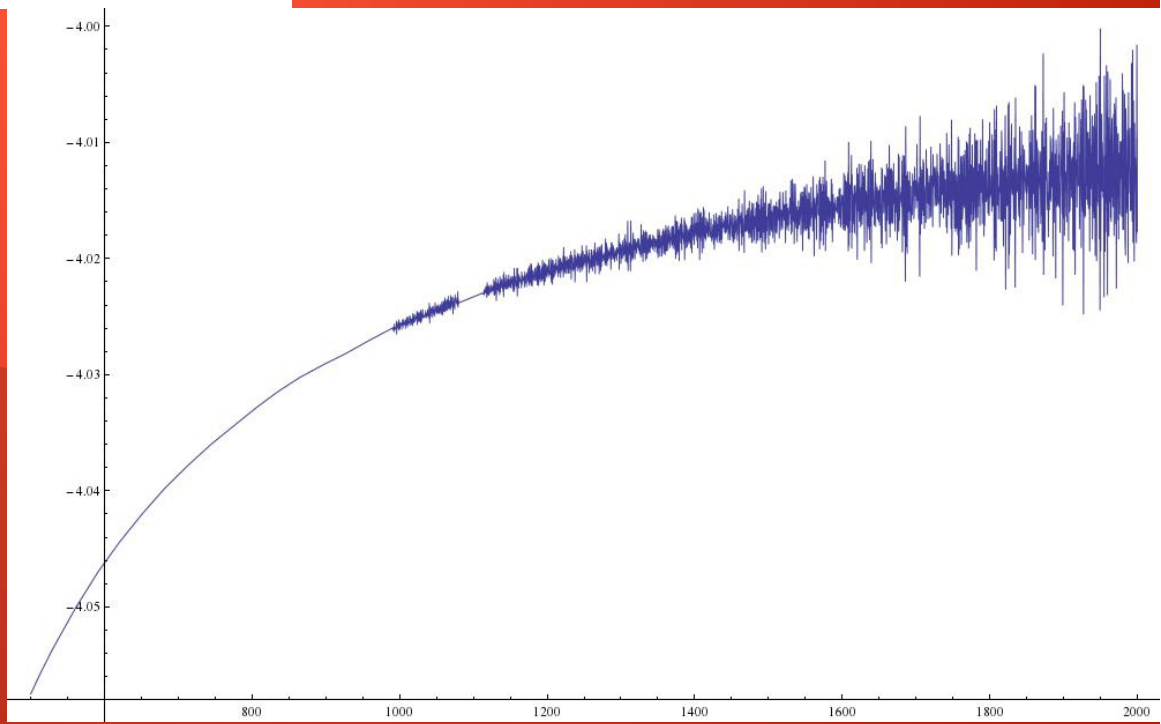
- Two primary measures of accuracy: quasinormal ringdown frequency and tail decay power
- The ringdown behavior is caused by a portion of the wave reflecting off of the curved space, then being re-reflected many times.
- During the tail behavior, the amplitude decreases as t^{-p}
- Eventually, the signal is dominated by numerical rounding error
- For $\alpha = 0.9$, the frequencies produced by the coupled and uncoupled cases were about equally accurate
- For $\alpha = 0.99$, the frequency produced by the coupled case was about 100 times more accurate than the uncoupled case
- However, the decay power for l=0 was consistently 4
- The correct value should be around 3
- Cause unknown



Discussion

- The coupling seems to be working correctly
- The discrepancy between expected and measured decay power may be caused by numerical inaccuracy
- Supported by the fact that the decay for higher l modes shows large amounts of noise
- The program has fourth-order accuracy
- Higher accuracy may be needed to correctly model the power law decay
- Possible topic for future investigation


Scalar Field Amplitude
1
1e-05
1e-10
1e-15
1e-20
1e-25
0
100
200
300
400
500
Time (M)
l = 0
l = 2
l = 4


-4.00
-4.01
-4.02
-4.03
-4.04
-4.05
800
1000
1200
1400
1600
1800
2000