Solving For Null Geodesics in Curved Spacetime

Bria Morgan[1], Dr. Peter Diener[2], Dr. Oleg Korobkin[2]

[1]University of Wisconsin – Eau Claire, WI, [2]Center for Computation & Technology, Louisiana State University, Baton Rouge, LA

Introduction

New, more sensitive astronomical instruments, such as the Very Large Baseline Array (VLBA), are expected to be implemented within the next decade. The unprecedented precision of their measurement capabilities will open up new possibilities of examining the sky near the supermassive black hole at the center of our galaxy and allow for new tests of general relativity. Currently there have only been a handful of experimental confirmations of general relativity, which only address the weak field regime. Numerical computations and simulations of relativistic equations allow us to understand the results and predictions of the theory of general relativity. Comparing these numerical predictions to observations can guide our interpretation of the data we receive, allow us to reconstruct the features of astronomical objects, and ultimately to test general relativity in the strong field regime.

Tests in Flat Space

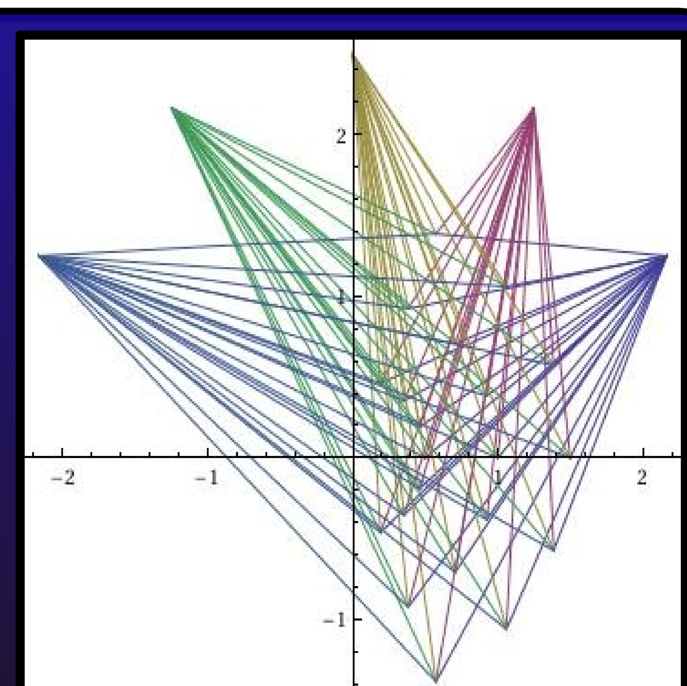


Fig. 1 Moving viewpoint in the x-y plane in flat space

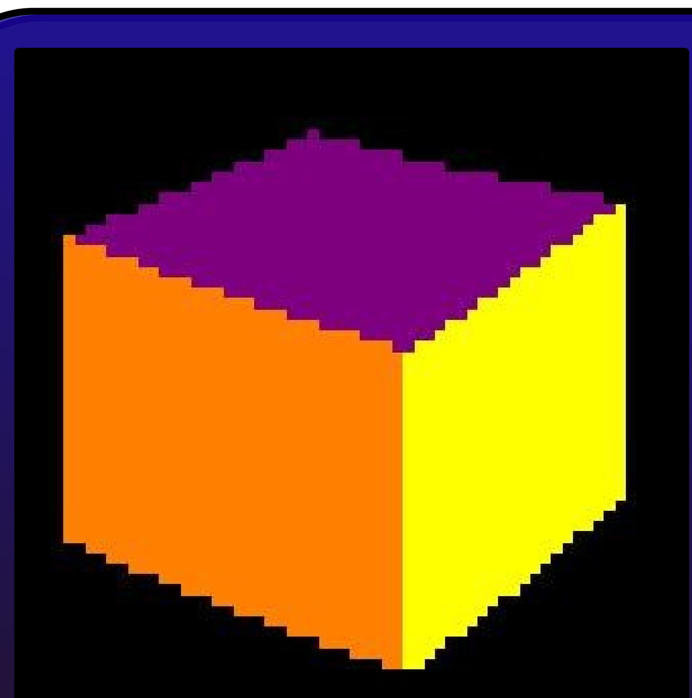


Fig. 2 Image of a cube in flat space generated by solving for geodesics at every point

Graphic Examples

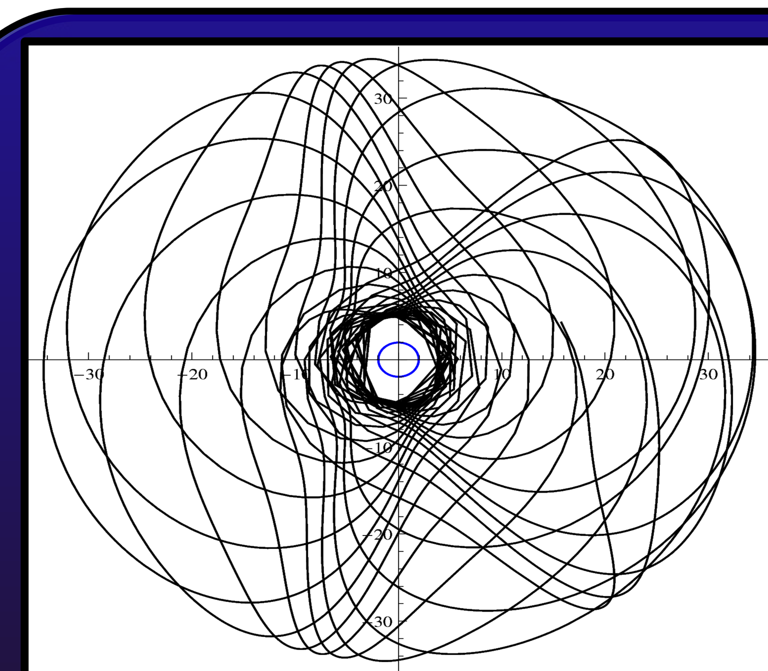


Fig. 3 Timelike orbit of a particle around a Kerr black hole. Event horizon shown in blue

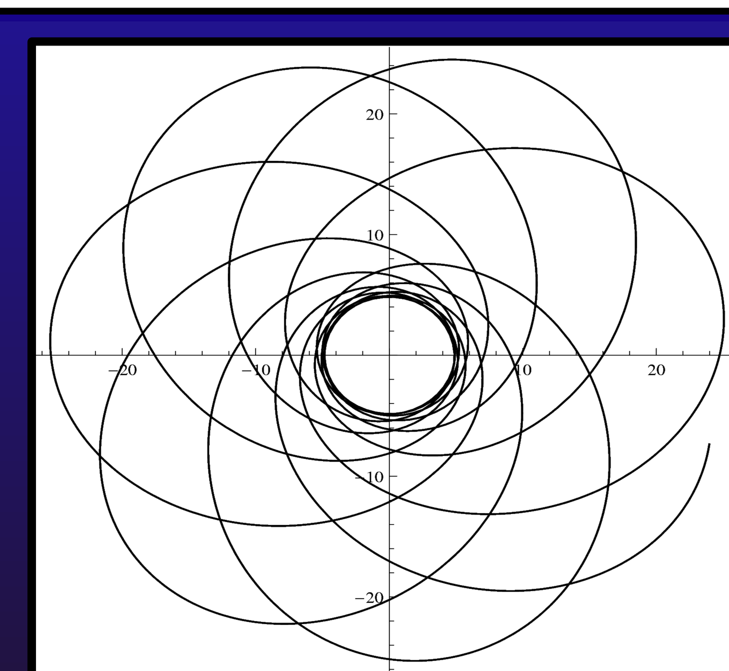


Fig. 4 Timelike orbit of a particle around a Schwarzschild black hole

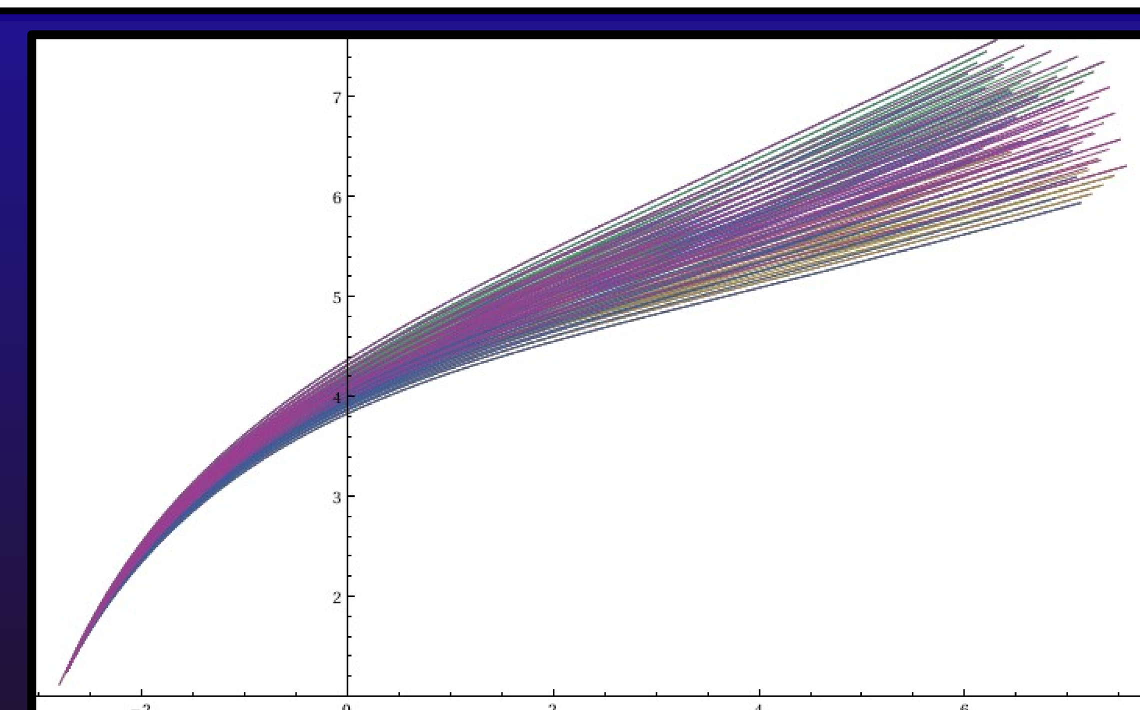
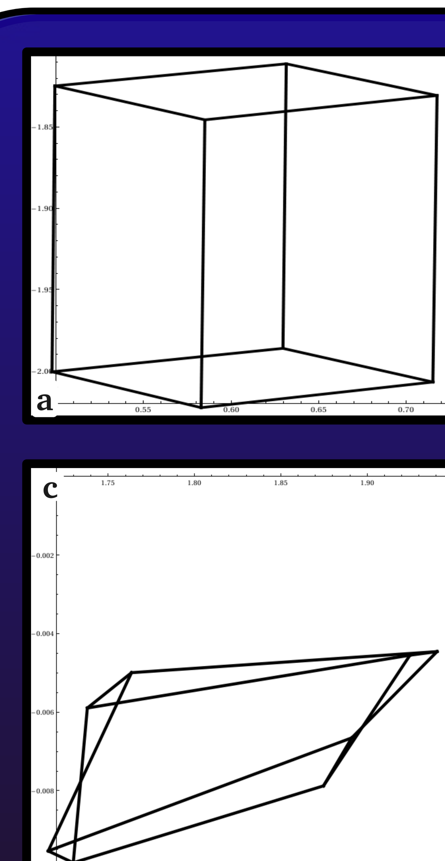


Fig. 5 Paths in the x-y plane of many photons originating from a single point near a Schwarzschild black hole



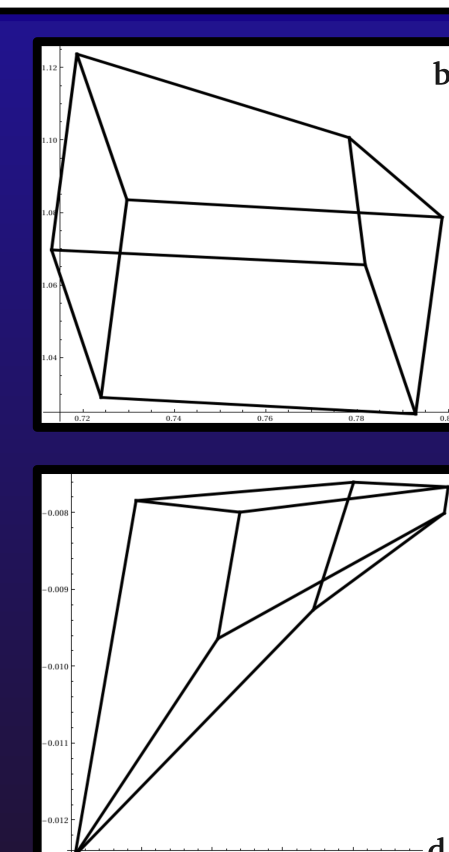


Fig. 6 Distortion of the image of a cube as viewpoint moves around a Schwarzschild black hole a) Undistorted reference cube b) Viewed from an angle of $\frac{\pi}{2}$ and below c) and d) Viewed from points on the opposite side of the black hole

Methods

For this project, we used Mathematica to derive the geodesic equations in explicit form.

$$\frac{d^2x^\alpha}{d\varphi^2} = \left(\Gamma^\varphi_{\beta\gamma}\frac{dx^\alpha}{d\varphi} - \Gamma^\alpha_{\beta\gamma}\right)\frac{dx^\beta}{d\varphi}\frac{dx^\gamma}{d\varphi}$$

The above version of the geodesic equation uses the coordinate φ as a parameter. When the following constraint is imposed on it,

$$g_{\mu\nu}v^\mu v^\nu = 0$$

it describes a null geodesic.

We then adapted NDSolve to find solutions for these equations using two different methods. The first method poses the problem as an initial value problem, with a starting point and direction. This method was used to trace a single geodesic path over time (Figures 3 and 4), and to generate solutions at every point in an image plane (Figure 2).

The second method solves the equations as a boundary value problem, initializing it with a starting and ending point, along with an estimation of the direction of the path between them. We added modifications to address issues involved in solving these equations, such as coordinate singularities, generating the initial guess, and the computational problem of a path that enters the singularity. Using them, we can graph traces of the geodesic paths (Figures 1 and 5). We also created a preliminary visualization programs that creates a projection of the image of an object in curved space (Figure 6).

Future Work

These functions are generalizable to different coordinate systems and spaces. Further work could streamline the transition between coordinates in an improved user interface. Other directions left to explore include:

- •Dynamical black holes
- •Expanding visualization capabilities
- •Porting code into Cactus

References

Schutz B F. A first course in general relativity. New York: Cambridge University Press; 2009.

Wolfram Research, Inc., Mathematica, Version 8.0, Champaign, IL; 2010.

Acknowledgements

This material is based upon work supported by the National Science Foundation under Grant OCI-1005165.